MTH 201: Multivariable Calculus and Differential Equations

Homework VII

(Due 5/11)

1. Convert the following Cartesian integrals into equivalent polar integrals and then evaluate them.

   (a) $\displaystyle\int_0^2\int_0^{\sqrt{4-y^2}} (x^2+y^2)\,dx\,dy$

   (b) $\displaystyle\int_0^1\int_x^{\sqrt{2-x^2}} (x+2y)\,dy\,dx$

   (c) $\displaystyle\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$

   (d) $\displaystyle\int_1^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}} e^{\sqrt{x^2+y^2}}\,dx\,dy$

   (e) $\displaystyle\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$

   (f) $\displaystyle\int_0^\infty\int_0^\infty \frac{1}{(1+x^2+y^2)^2}\,dx\,dy$

2. Integrate $f(x,y)=\dfrac{\ln(x^2+y^2)}{\sqrt{x^2+y^2}}$ over the region $1\le x^2+y^2\le e^2$.

3. Convert the integral
$$\int_{-1}^1\int_0^{\sqrt{1-y^2}}\int_0^x (x^2+y^2)\,dz\,dx\,dy$$
into an equivalent integral in cylindrical coordinates and evaluate.

4. Set up the interated integral for evaluating $\iiint_D f(r,\theta,z)\,r dz\,dr\,d\theta$ over the given region D.

   (a) D is the right circular cylinder whose base is the circle $r=3\cos\theta$ and top lies in the plane $z=5-x$.

   (b) D is the solid right cylinder whose base is the region between the circles $r=\cos\theta$ and $r=2\cos\theta$, and whose top lies in $z=3-y$.

   (c) D is prism whose base is the triangle in the xy-plane bounded by the y-axis and the lines $y=x$ and $y=1$, and whose top lies in the plane $z=2-x$.

5. Find the spherical coordinate limits for the integral that calculates the volume of the given solid or region, and then evaluate the integral.

   (a) The solid bounded below by $\rho=2\cos\phi$ and above by the cone $z=\sqrt{x^2+y^2}$.

   (b) The solid bounded below by the xy-plane, on the sides by the sphere $\rho=2$, and above by the cone $\phi=\pi/3$.

   (c) The solid enclosed by the cone $z=\sqrt{x^2+y^2}$ between the planes $z=1$ and $z=2$.

   (d) The region bounded below by the paraboloid $z=x^2+y^2$, laterally by the cylinder $x^2+y^2=1$, and above by the paraboloid $z=x^2+y^2+1$.

   (e) The region cut from the solid cylinder $x^2+y^2\le 1$ by the sphere $x^2+y^2+z^2=4$.

   (f) The region enclosed by the cylinder $x^2+y^2=4$ and planes $z=0$ and $y+z=4$.